Dimension 2 Condensate in Large N_c Regge Models

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Dimension 2 condensates ?

Standard wisdom: There are no local gauge invariant operators of dimension 2

But , are there operators of dimension 2 relevant for phenomenology ?

There exists a nonlocal gauge invariant condensate,

$$\langle A^2_{\rm min} \rangle = \frac{1}{VT} {\rm min}_g \int d^4x \langle \left(g A_\mu g^\dagger + g \partial_\mu g^\dagger \right)^2 \rangle, \qquad (1)$$

which reduces to the $\langle A^2 \rangle$ condensate in the Landau gauge.

Example: Can f_π^2 be written as a the expectation value of a gauge invariant local operator ?

In perturbation theory one must fix the gauge. Then one breaks gauge invariance and in the quantum theory the "memory" of gauge invariance is BRST symmetry.

There is no complete gauge fixing. Thus, there is always a dimension-2 local operator which is invariant under the residual gauge invariance.

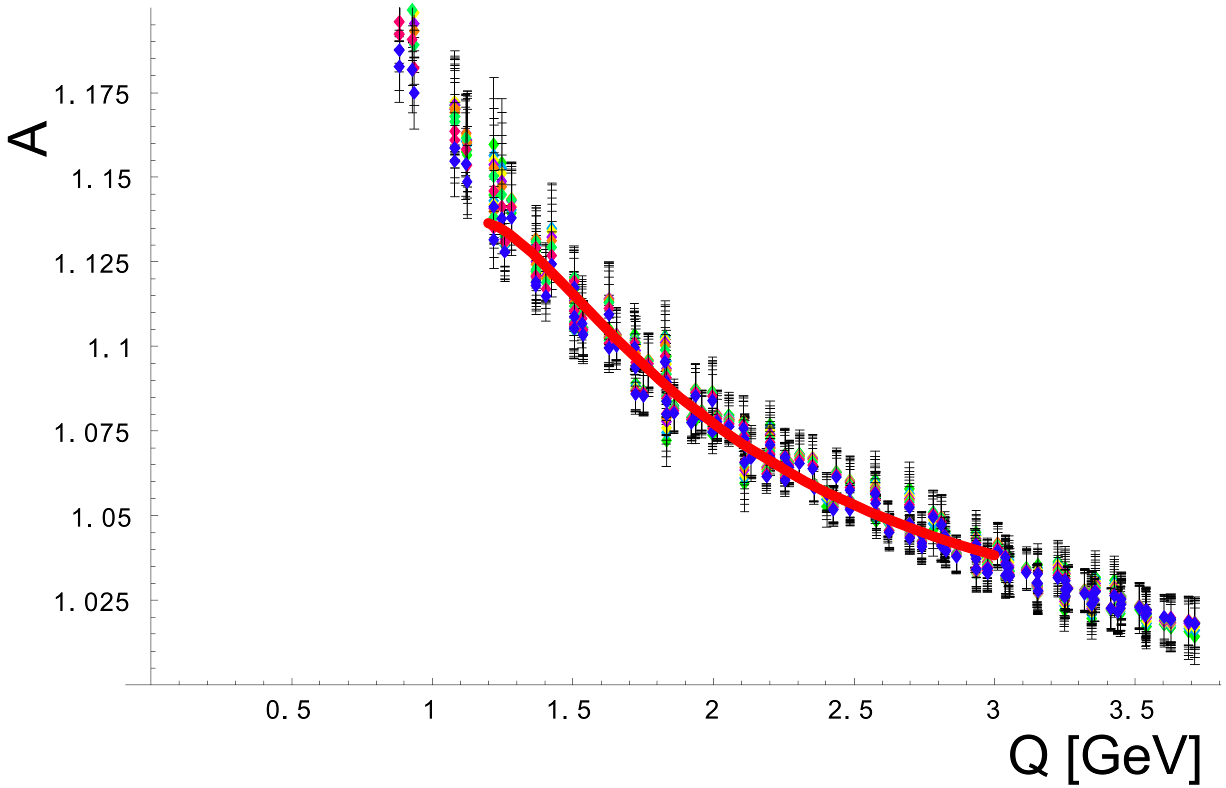
Dimension-2 objects have been seen in fixed gauge lattices: (See Talks by Skullerad, Rodriguez-Qintero, Oliveira)

The dynamical origin of Dimension 2 condensates is unclear

Quark propagator on the Lattice in Landau Gauge

P. Bowman, W. Broniowski, E. R. A.
Phys.Rev.D70:097505,2004

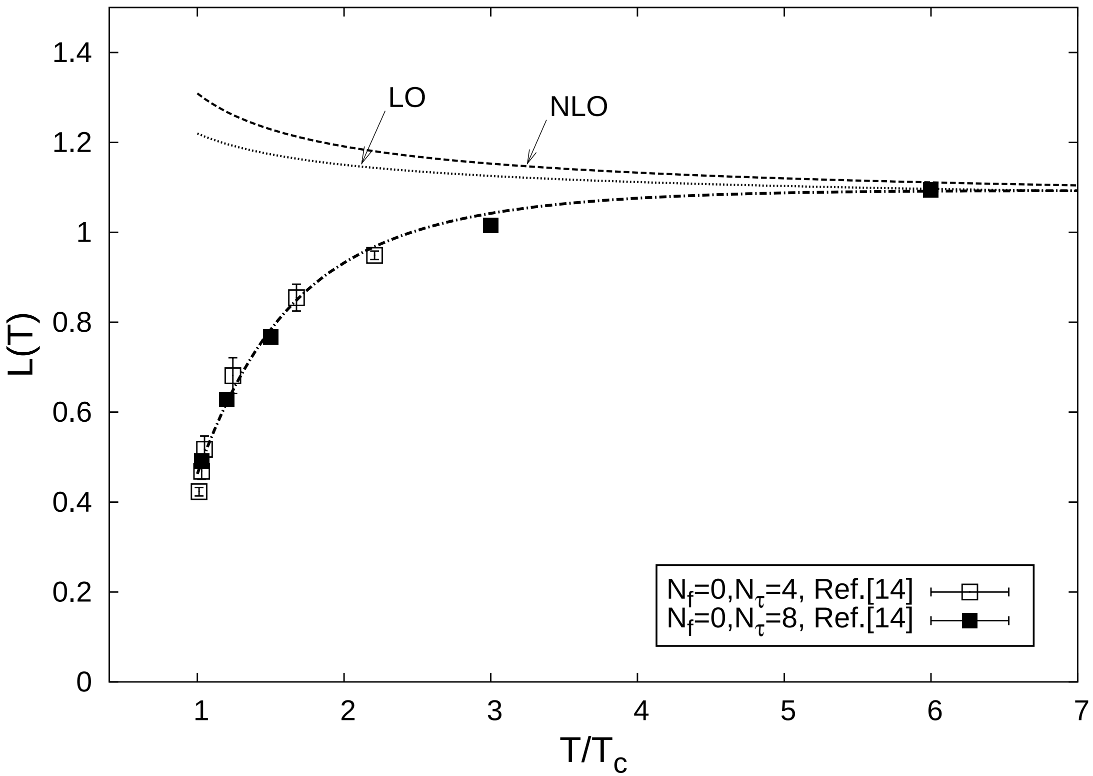
$$A(Q) \;=\; 1+\frac{\pi\alpha_s(\mu^2)\langle A^2\rangle_\mu}{N_c Q^2}-\frac{\pi\alpha_s(\mu^2)\langle G^2\rangle_\mu}{3N_c Q^4}+\frac{3\pi\alpha_s(\mu^2)\langle\bar q g \not{A} q\rangle_\mu}{4Q^4},$$



The Polyakov loop above the deconfinement phase transition,

(see E. Megias, L.L. Salcedo and E.R.A. JHEP 2006)

$$L = \left\langle \frac{1}{N_c} \mathrm{Tr}_c e^{iA_0/T} \right\rangle \to e^{-\langle A_0^2 \rangle / 2 N_c T^2}$$



Standard wisdom: Operator Product Expansion

$$
\begin{aligned}
\Pi_V^{\mu a,\nu b}(q) &= i\int d^4x e^{-iq\cdot x}\langle 0|T\left\{J_V^{\mu a}(x)J_V^{\nu b}(0)\right\}|0\rangle \\
&= \Pi_V^T(q^2)\,\left(q^\mu q^\nu - g^{\mu\nu}q^2\right)\delta^{ab}, \\
\Pi_A^{\mu a,\nu b}(q) &= i\int d^4x e^{-iq\cdot x}\langle 0|T\left\{J_A^{\mu a}(x)J_A^{\nu b}(0)\right\}|0\rangle \\
&= \Pi_A^T(q^2)\,\left(q^\mu q^\nu - g^{\mu\nu}q^2\right)\delta^{ab} + \Pi_A^L(q^2)\,q^\mu q^\nu \delta^{ab},
\end{aligned}
$$

with $J_{V,A}^{\mu a} = \bar\psi i\gamma^\mu\{1,\gamma_5\}\frac{\tau^a}{2}\psi$ QCD currents. At large Q

$$
\begin{aligned}
\Pi_{V+A}^T(Q^2) &= \frac{1}{4\pi^2}\left\{-\frac{N_c}{3}\log\frac{Q^2}{\mu^2} + \frac{\pi}{3}\frac{\langle\alpha_S G^2\rangle}{Q^4} + \frac{256\pi^3}{81}\frac{\alpha_S\langle\bar q q\rangle^2}{Q^6}\right\} + \dots, \\
\Pi_{V-A}^T(Q^2) &= -\frac{32\pi}{9}\frac{\alpha_S\langle\bar q q\rangle^2}{Q^6} + \dots,
\end{aligned} \tag{2}
$$

Coordinate space

$$\langle V_\mu(x) V_\nu(0) \rangle = A(x) g^{\mu\nu} + B(x) x^\mu x^\nu$$

Current conservation implies that one has one independent function

$$\langle V_\mu(x) V_\nu(0) \rangle = \left(g^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu \right) \Pi(x)$$

The function $\Pi(x)$ has dimension x^{-4}. Thus, at short distances one expects to have (up to possible logarithms)

$$\Pi(x) = \frac{O_0}{x^4} + \frac{O_2}{x^2} + \frac{O_4}{x^0} + \frac{O_6}{x^2} + \dots$$

The term containing an operator of dimension 2 is very special. It yields a contribution of the form

$$\left(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)\frac{1}{x^2} \;=\; \frac{g^{\mu\nu}x^2 - 4x^\mu x^\nu}{x^4}$$

which, in addition of being conserved, is also traceless.

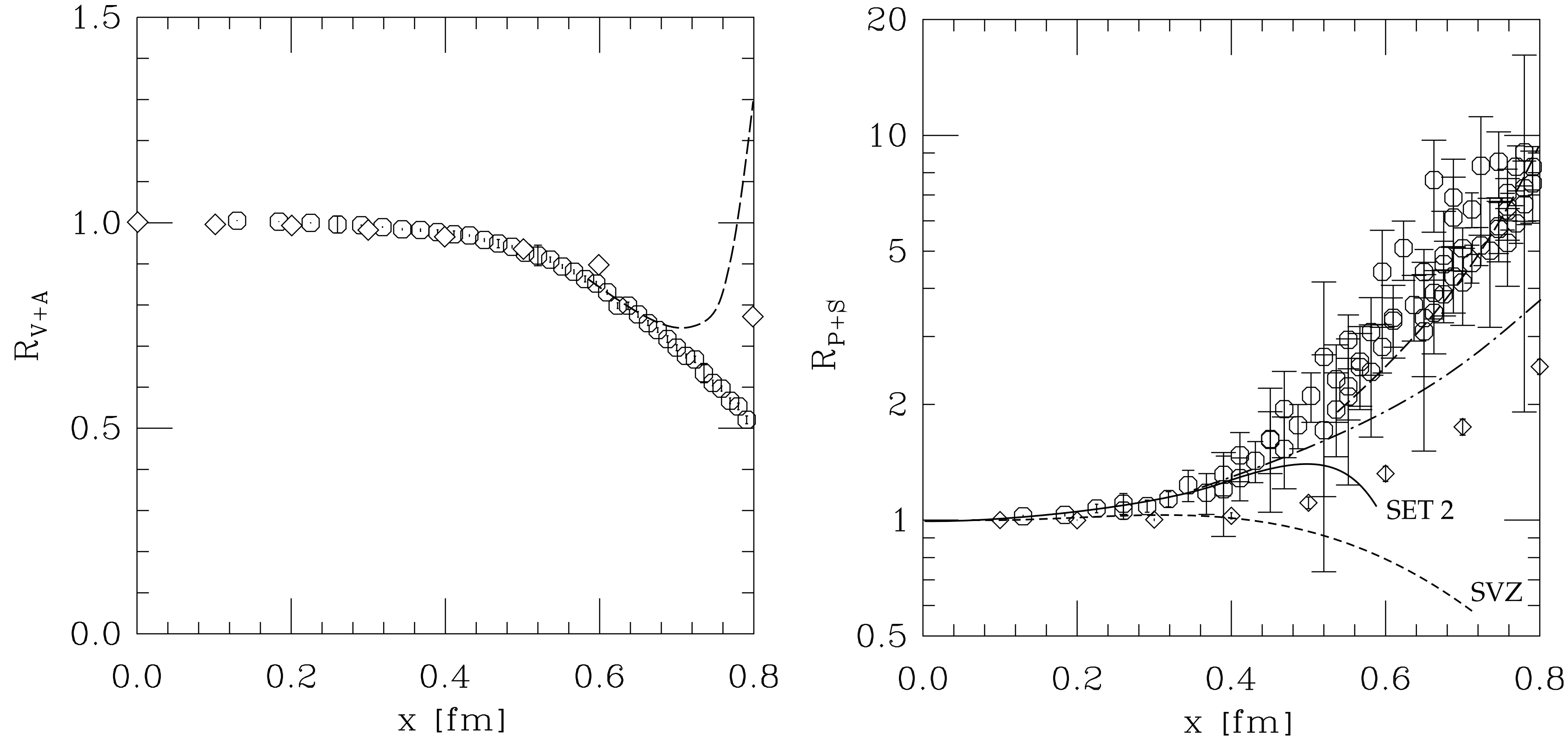
Thus, contracting and taking the derivative are non-commuting operations.

There is no traceless and conserved term in momentum space. Conservation implies the form $A(q^\mu q^\nu - q^2 g^{\mu\nu})$ but tracelessness requires $A = 0$.

This problem appears in chiral quark models : The dimension 2 object is the constituent quark mass squared !!

$$O_2 \sim \langle \sigma^2 + \vec{\pi}^2 \rangle \sim M^2$$

Narison+Zakharov (2002)



The unconventional OPE

Chetyrkin, Narison, Zakharov (1999)

$$\Pi^T_{V+A}(Q^2) \;=\; \frac{1}{4\pi^2}\Big\{ -\frac{N_c}{3}\log\frac{Q^2}{\mu^2} - \frac{\alpha_S}{\pi}\frac{\lambda^2}{Q^2} + \frac{\pi}{3}\frac{\langle\alpha_S G^2\rangle}{Q^4} + \frac{256\pi^3}{81}\frac{\alpha_S\langle\bar{q}q\rangle^2}{Q^6}\Big\} +$$

$$\Pi^T_{V-A}(Q^2) \;=\; -\frac{32\pi}{9}\frac{\alpha_S\langle\bar{q}q\rangle^2}{Q^6} + \dots\,,$$

the dimension-2 coefficient λ^2 is interpreted as the tachyonic gluon mass, m_g, providing the short-distance string tension

$$\sigma_0 = -2\alpha_s\lambda^2/N_c. \qquad (8)$$

The large N_c limit

In the large-N_c limit the vacuum sector of QCD becomes a theory of infinitely many non-interacting mesons and glueballs, hence the correlators may be saturated by infinitely many meson states. Thus one has, up to subtractions,

$$\begin{aligned}
\Pi_V^T(Q^2) &= \sum_{n=0}^{\infty} \frac{F_{V,n}^2}{M_{V,n}^2 + Q^2} + c.t., \\
\Pi_A^T(Q^2) &= \frac{f^2}{Q^2} + \sum_{n=0}^{\infty} \frac{F_{A,n}^2}{M_{A,n}^2 + Q^2} + c.t.,
\end{aligned} \tag{9}$$

where the first term in the axial-vector channel is the massless pion contribution with $f = 86\mathrm{MeV}$ denoting the pion decay constant in the chiral limit.

The Regge spectrum

Let us consider two scalar quarks, in the CM frame the mass operator is given by

$$M = 2\sqrt{p^2 + m^2} + \sigma r \qquad (10)$$

where σ is the string tension. Squaring the operator we get

$$\frac{1}{4}(M - \sigma r)^2 - m^2 = p^2 = p_r^2 + \frac{L^2}{r^2} \qquad (11)$$

This is a standard Schrödinger operator and for simplicity let us assume $L = 0$ and $m = 0$. The Bohr-Sommerfeld semiclassical quantization condition for the radial excitations reads

$$2\int_0^a p_r dr = 2\pi(n + \alpha) \qquad (12)$$

where the turning point is given by $M = \sigma a$ the spectrum is

$$M_n^2 = 2\pi\sigma(n+\alpha) \tag{13}$$

For large meson masses, the level density is given by

$$\begin{aligned} \rho(M^2) &= \sum_n \delta(M^2 - M_n^2) \to \int dn \delta(M^2 - M_n^2) \\ &= \frac{1}{dM_n^2/dn}\Big|_{M_n^2=M^2} = \frac{dn}{dM^2} = \frac{1}{2\pi\sigma} \end{aligned} \tag{14}$$

So, using the WKB approximation we get

$$\frac{dn}{dM^2} = \frac{1}{\pi}\int_0^a \frac{dr}{p_r} = \frac{1}{2\pi\sigma} \tag{15}$$

which is a constant. If we include finite quark mass corrections we get instead a level density

$$\frac{dn}{dM^2} = \frac{1}{2\pi\sigma}\sqrt{1-\frac{4m^2}{M^2}}\theta(M^2-4m^2) \tag{16}$$

Note that the factor is identical to the two body phase space factor which appears in the absortive part of two point correlators for FREE PARTICLES.

So at large energies the WKB approximation holds and looks like the phase space of two free particles.

Large N_c Regge models

We use radial Regge spectra to saturate vector and axial-vector channels,

$$M_{V,n}^2 = M_V^2 + a_V n,\ M_{A,n}^2 = M_A^2 + a_A n,\ n = 0, 1, \ldots \quad (17)$$

which is well fulfilled in the experimentally explored region.

The vector correlator satisfies the once-subtracted dispersion relation, hence

$$\Pi_V^T(Q^2) \;=\; \sum_{n=0}^{\infty}\left(\frac{F_{V,n}^2}{M_V^2 + a_V n + Q^2} - \frac{F_{V,n}^2}{M_V^2 + a_V n}\right). \quad (18)$$

The sum needs to reproduce the $\log Q^2$ term of the OPE expansion for which only the asymptotic part of the spectrum matters.

At large n the behavior of the residues is $F_{V,n} \simeq F_V$, *i.e.* there is no n-dependence. Similarly, $F_{A,n} \simeq F_A$.

We also assume $a_V = a_A$, which means that for asymptotic n

$$F_V = F_A = F, \quad a_V = a_A = a = 2\pi\sigma. \tag{19}$$

$a_V = a_A$ yields the same density of states in the V and A channels which complies to the "chiral symmetry restoration" at high energies.

Strict Regge model

$$\Pi^T_{V-A}(Q^2) = \frac{F^2}{a}\left[-\psi\left(\frac{M_V^2+Q^2}{a}\right)+\psi\left(\frac{M_A^2+Q^2}{a}\right)\right]$$

$$-\quad \frac{f^2}{Q^2} \simeq \Pi^T_{V-A}(Q^2) = \left(\frac{F^2}{a}(M_A^2-M_V^2)-f^2\right)\frac{1}{Q^2}$$

$$+\left(\frac{F^2}{2a}(M_A^2-M_V^2)(a-M_A^2-M_V^2)\right)\frac{1}{Q^4}+\ldots \qquad (20)$$

Matching to (7) yields the two Weinberg sum rules:

$$f^2 = \frac{F^2}{a}(M_A^2-M_V^2), \qquad \text{(WSR I)}$$

$$0 = (M_A^2-M_V^2)(a-M_A^2-M_V^2). \quad \text{(WSR II)}$$

$$\Pi^T_{V+A}(Q^2) = \frac{F^2}{a}\left[-\psi\left(\frac{M_V^2+Q^2}{a}\right) - \psi\left(\frac{M_A^2+Q^2}{a}\right)\right]$$
$$+\frac{f^2}{Q^2} + const \simeq -\frac{2F^2}{a}\log\frac{Q^2}{\mu^2}$$
$$+\left(f^2 + F^2 - \frac{F^2}{a}(M_A^2 + M_V^2)\right)\frac{1}{Q^2} \quad (21)$$
$$+\frac{F^2}{6a}\left(a^2 - 3a(M_A^2+M_V^2) + 3(M_A^4+M_V^4)\right)\frac{1}{Q^4} + \dots$$

The integration constant has been absorbed in the scale μ. Matching of the coefficient of the $\log Q^2$ to (7) gives

$$a = 2\pi\sigma = 24\pi^2 F^2/N_c, \quad (22)$$

where σ is the (long-distance) string tension.

$$\begin{aligned} M_A^2 &= M_V^2 + \frac{24\pi^2}{N_c} f^2, \\ a &= M_A^2 + M_V^2 = 2M_V^2 + \frac{24\pi^2}{N_c} f^2, \end{aligned} \qquad (23)$$

and

$$\begin{aligned} -\frac{\alpha_S \lambda^2}{4\pi^3} &= f^2, \qquad (24) \\ \frac{\alpha_S \langle G^2 \rangle}{12\pi} &= \frac{M_A^4 - 4M_V^2 M_A^2 + M_V^4}{48\pi^2} \\ &= \frac{288\pi^4 f^4/N_c^2 - 24\pi^2 f^2 M_V^2/N_c - M_V^4}{24\pi^2}. \end{aligned}$$

The numerical value for the dimension-2 condensate is $-\frac{\alpha_S \lambda^2}{\pi} = 0.3\mathrm{GeV}^2$ as compared to the value of $0.12\mathrm{GeV}^2$ from Chetyrkin, Narison and Zakharov.

Andreev (2006) using AdS/CFT also quotes the estimate $-\frac{\alpha_S \lambda^2}{\pi} = 0.3\mathrm{GeV}^2$.

We get $\sqrt{\sigma_0} = 782\mathrm{MeV}$, about twice as much, thus the consistency check is violated badly.

Also, the dimension-4 gluon condensate is negative for $M_V \geq 0.46$ GeV. Actually it never, not even at very low values of M_V, reaches the QCD sum-rules value of the condensate. The dimension-6 condensate in the model is zero in the $V + A$ channel, while from OPE it should not be. All these problems show that the strictly linear radial Regge model with constant residues is *too restrictive*.

Modified Regge model

For the purpose of illustration we consider the following simple modification of the previous model:

$$\begin{aligned} M_{V,0} &= m_\rho,\ M^2_{V,n} = M^2_V + an, \quad n \geq 1, \\ M^2_{A,n} &= M^2_A + an, \quad n \geq 0. \end{aligned} \tag{25}$$

In words, the lowest ρ mass is shifted, otherwise all is kept "universal", including constant residues for all states. In the present case the Weinberg sum rules have the form (we set $N_c = 3$ from now on)

$$\begin{aligned} M^2_A &= M^2_V + 8\pi^2 f^2, \\ a &= 8\pi^2 F^2 = \frac{8\pi^2 f^2 \left(4\pi^2 f^2 + {M_V}^2\right)}{4\pi^2 f^2 - {m_\rho}^2 + {M_V}^2}. \end{aligned} \tag{26}$$

When $m_\rho = 0.77\mathrm{GeV}$ is fixed, the model has only one free parameter left. We may take it to be M_V, however, it is more convenient to express it through the string tension σ, which is then treated as a free parameter. Thus

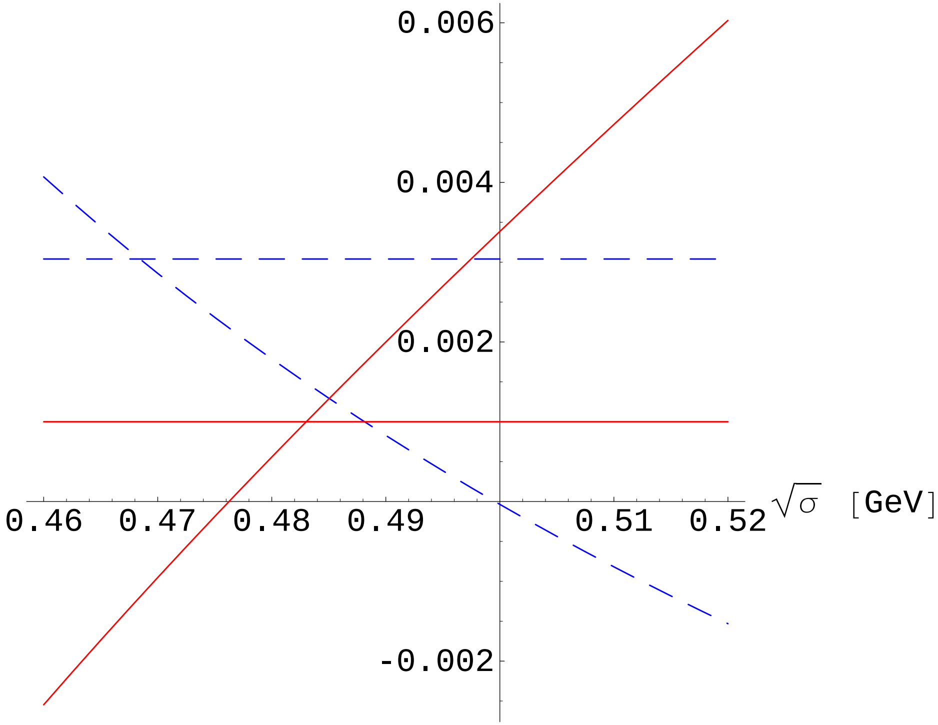
$$M_V^2 = \frac{-16\pi^3 f^4 + 4\pi^2\sigma f^2 - {m_\rho}^2\sigma}{4f^2\pi - \sigma}, \tag{27}$$

and the gluon condensates, obtained by matching to (7), are

$$\begin{aligned} -\frac{\alpha_S\lambda^2}{4\pi^3} &= \frac{16\pi^3 f^4 - \pi\sigma^2 + {m_\rho}^2\sigma}{16f^2\pi^3 - 4\pi^2\sigma} \\ \frac{\alpha_S\langle G^2\rangle}{12\pi} &= 2\pi^2 f^4 - \pi\sigma f^2 \\ &+ \frac{3\sigma\left(\frac{{m_\rho}^2\sigma}{\left(\sigma-4f^2\pi\right)^2} - 2\pi\right){m_\rho}^2}{8\pi^2} + \frac{\sigma^2}{12}. \end{aligned} \tag{28}$$

The constant lines mark the "physical" values $-\alpha_s\lambda^2/(4\pi^3) = 0.003\mathrm{GeV}^2$ and $\alpha_S\langle G^2\rangle/(12\pi) = 0.001\mathrm{GeV}^4$.

The consistency check $\sigma = \sigma_0$ is satisfied for $\sqrt{\sigma} = 497\mathrm{MeV}$.



Other predictions

We apply the Das-Mathur-Okubo sum rule to evaluate the low-energy constant L_{10}, and the Das-Guralnik-Mathur-Low-Yuong sum rule to obtain the electromagnetic pion mass splitting.

In the strictly linear Regge model with, for instance, $M_A^2 = 2M_V^2$ and $M_V = \sqrt{24\pi^2/N_c} f = 764\mathrm{MeV}$, we have $a = 3M_V^2$, or $\sqrt{\sigma} = \sqrt{3/2\pi} M_V = 532\mathrm{MeV}$, and $F = \sqrt{3} f = 150\mathrm{MeV}$, rather reasonable results.

Analytically,

$$L_{10} = \frac{F^2}{4a}\left[\psi\left(\frac{M_V^2}{a}\right) - \psi\left(\frac{M_A^2}{a}\right)\right], \qquad (29)$$

while the electromagnetic mass difference involves a numerical integral.

Then in the strict Regge model

$$L_{10} = -N_c/(96\sqrt{3}\pi) = -5.74 \times 10^{-3}(-5.5 \pm 0.7 \times 10^{-3})_{\rm exp}$$

and

$$m^2_{\pi^\pm} - m^2_{\pi_0} = (31.4{\rm MeV})^2 \ (35.5{\rm MeV})^2_{\rm exp}.$$

In the Regge modified model $\sigma = (0.48{\rm GeV})^2$ the values are

$$L_{10} = -5.2 \times 10^{-3}$$

and

$$m^2_{\pi^\pm} - m^2_{\pi_0} = (34.4{\rm MeV})^2,$$

in quite remarkable agreement with the data.

Conclusions

The scheme of matching OPE to the radial Regge models produces, in a natural way, the $1/Q^2$ correction to the vector and axial vector correlators, which is attributed to the dimension-2 gluon condensate.

Our explicit calculation illustrates the significance of confinement also for the short-distance expansion.

More generally, OPE with the dimension-2 and all other condensates can be matched by radial Regge models, provided conditions are satisfied by the asymptotic spectra, and the parameters of the low-lying states are adjusted to reproduce the values of the condensates.

In principle, these parameters are measurable, hence the information encoded in the low-lying states is the same as the information in the condensates and we could verify consistency.

Yet the sensitivity of the values of the condensates to the parameters of the spectra, as seen by comparing the two explicit models considered in this paper, make such a study difficult at a more precise level.